Does the present data on $B_s - \bar{B}_s$ mixing rule out a large enhancement in the branching ratio of $B_s \to \mu^+\mu^-$?

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In this letter, we consider the constraints imposed by the recent measurement of $B_s - \bar{B}_s$ mixing on the new physics contribution to the rare decay $B_s \to \mu^+\mu^-$. New physics in the form vector and axial-vector couplings is already severely constrained by the data on $B \to (K, K^*)\mu^+\mu^-$. Here, we show that $B_s - \bar{B}_s$ mixing data, together with the data on $K^0 - \bar{K}^0$ mixing and $K_L \to \mu^+\mu^-$ decay rate, strongly constrain the scalar-pseudoscalar contribution to $B_s \to \mu^+\mu^-$. We conclude that new physics *can at best* lead to a factor of 2 increase in the branching ratio of $B_s \to \mu^+\mu^-$ compared to its Standard Model expectation.

The flavour changing neutral interaction (FCNI) $b \to s\mu^+\mu^-$ serves as an important probe to test the Standard Model (SM) and its possible extensions. This four fermion interaction gives rise to semi-leptonic decays $B \to (K, K^*)\mu^+\mu^-$ and also the purely leptonic decay $B_s \to \mu^+\mu^-$. The semi-leptonic decays $B \to (K, K^*)\mu^+\mu^-$ have been observed experimentally [1, 2, 3] with branching ratios close to their SM predictions [4, 5, 6]. At present there is only an upper limit, 1.0×10^{-7} at 95% C.L., on the branching ratio of the decay $B_s \to \mu^+\mu^-$ [7, 8]. The SM prediction for this branching ratio is $(3.2 \pm 1.5) \times 10^{-9}$ [9] or $\leq 7.7 \times 10^{-9}$ at 3σ level. $B_s \to \mu^+\mu^-$ will be one of the important rare B decays to be studied by the experiments at the upcoming Large Hadron Collider (LHC). We expect that the present upper limit will be reduced significantly in these experiments. A non-zero value of this branching ratio is measurable, if it is $\geq 10^{-8}$ [10].

In a previous publication [11], we studied the constraints on new physics contribution to the branching ratio of $B_s \to \mu^+\mu^-$ coming from the experimentally measured values of the branching ratios of $B \to (K, K^*)\mu^+\mu^-$. We found that if the new physics interactions are in the form of vector/axial-vector operators, then the present data on $B(B \to (K, K^*)\mu^+\mu^-)$ does not allow a large boost in $B(B_s \to \mu^+\mu^-)$. By large boost we mean an enhancement of at least an order of magnitude in comparison to the SM prediction. However, if the new

physics interactions are in the form of the scalar/pseudoscalar operators, then the presently measured rates of $B \to (K, K^*)\mu^+\mu^-$ do not put any useful constraints on $B_s \to \mu^+\mu^-$ and $B_{NP}(B_s \to \mu^+\mu^-)$ can be as high as the present experimental upper limit. Therefore we are led to the conclusion that if future experiments measure $B_s \to \mu^+\mu^-$ with a branching ratio greater than 10^{-8}, then the new physics giving rise to this decay has to be in the form of scalar/pseudoscalar interaction.

Recently $B_s - \bar{B}_s$ mixing has been observed experimentally [13], with a very small experimental error. In this paper, we want to see what constraint this measurement imposes on the new physics contribution to the branching ratio of $B_s \to \mu^+\mu^-$. In particular, we consider the question: Does it allow new physics in the form of scalar/pseudoscalar interaction to give a large boost in $B_{NP}(B_s \to \mu^+\mu^-)$?

We start by considering the $B_s \to \mu^+\mu^-$ decay. The effective new physics lagrangian for the quark level transition $\bar{b} \to \bar{s}\mu^+\mu^-$ due to scalar/pseudoscalar interactions can arise from tree and/or electroweak penguin and/or box diagrams. We parametrize it as

$$L^{SP}_{\bar{b}\to\bar{s}\mu^+\mu^-} = G_1\, \bar{b}(g_S^{sb} + g_P^{sb}\gamma_5)s\ \bar{\mu}(g_S^{\mu\mu} + g_P^{\mu\mu}\gamma_5)\mu, \tag{1}$$

where G_1 is a dimensional factor characterizing the overall scale of new physics, with dimension $(mass)^{-2}$. This factor essentially arises due to the scalar propagator in tree or electroweak penguin diagrams (or scalar propagators in box diagrams) which couples the quark bilinear to the lepton bilinear. $g^{sb}_{S,P}$ and $g^{\mu\mu}_{S,P}$ are dimensionless numbers, characterizing, respectively, $b - s$ and $\mu - \mu$ couplings due to new physics scalar/pseudoscalar interactions. Electromagnetic penguins necessarily have vector couplings in the lepton bilinear so they do not contribute to the effective lagrangian in eq. (1). The amplitude for the decay $B_s \to l^+l^-$ is given by

$$M(B_s \to \mu^+\mu^-) = G_1\, g_P^{sb}\langle 0 \left|\bar{b}\gamma_5 s\right| B_s\rangle \left[g_S^{\mu\mu}\bar{u}(p_\mu)v(p_{\bar{\mu}}) + g_P^{\mu\mu}\bar{u}(p_\mu)\gamma_5 v(p_{\overline{\mu}})\right]. \tag{2}$$

The pseudoscalar matrix element is,

$$\langle 0 \left|\bar{b}\gamma_5 s\right| B_s\rangle = -i\frac{f_{B_s}M^2_{B_s}}{m_b + m_s}, \tag{3}$$

where m_b and m_s are the masses of bottom and strange quark respectively.

The calculation of the decay rate gives

$$\Gamma_{NP}(B_s \to \mu^+\mu^-) = (g_P^{sb})^2[(g_S^{\mu\mu})^2 + (g_P^{\mu\mu})^2]\frac{G_1^2}{8\pi}\frac{f^2_{B_s}M^5_{B_s}}{(m_b + m_s)^2}. \tag{4}$$

We see that the decay rate depends upon the new physics couplings $(g_P^{sb})^2$ and $G_1^2[(g_S^{\mu\mu})^2 + (g_P^{\mu\mu})^2]$. To obtain information on these parameters, we look at $B_s - \bar{B}_s$ mixing together with $K_L \to \mu^+\mu^-$ decay and $K^0 - \bar{K^0}$ mixing.

Let us consider $B_s - \bar{B}_s$ mixing to obtain a constraint on $(g_P^{sb})^2$. Replacing leptonic bilinear by quark bilinear in eq. 1, we get $\Delta B = 2$ Lagrangian,

$$L^{SP}_{B_s - \bar{B}_s} = G_2\, \bar{b}(g_S^{sb} + g_P^{sb}\gamma_5)s\ \bar{b}(g_S^{sb} + g_P^{sb}\gamma_5)s, \tag{5}$$

where G_2 is another dimensional factor. As in the case of G_1, introduced in eq. (1), G_2 also arises due to the scalar propagator (or progators in the case of box diagrams). Therefore it also has dimension $(mass)^{-2}$ and *is of the same order of magnitude as* G_1. From eq. (5), we calculate the mass difference of the B_s mesons to be

$$\Delta m_{B_s} = \frac{1}{2M_{B_S}} G_2\, (g_P^{sb})^2 \hat{B}_{B_s} \frac{f_{B_s}^2 M_{B_s}^4}{(m_b + m_s)^2}. \tag{6}$$

Thus the effective $b - s$ pseudoscalar coupling is obtained to be

$$(g_P^{sb})^2 = \frac{\Delta m_{B_s}(m_b + m_s)^2}{2\hat{B}_{B_s} f_{B_s}^2 M_{B_s}^3 G_2}. \tag{7}$$

We now consider the decay $K_L \to \mu^+\mu^-$. The same new physics leading to the effective $\bar{b} \to \bar{s}\mu^+\mu^-$ lagrangian in eq. (1), also leads a similar effective lagrangian for $\bar{s} \to \bar{d}\mu^+\mu^-$ transition. The only difference will be the effective scalar/pseudoscalar couplings in the quark bilinear. Thus we have,

$$L^{SP}_{\bar{s} \to \bar{d}\mu^+\mu^-} = G_1\, \bar{s}(g_S^{sd} + g_P^{sd}\gamma_5)d\, \bar{\mu}(g_S^{\mu\mu} + g_P^{\mu\mu}\gamma_5)\mu. \tag{8}$$

The calculation of decay rate gives

$$\Gamma_{NP}(K_L \to \mu^+\mu^-) = 2(g_P^{sd})^2[(g_S^{\mu\mu})^2 + (g_P^{\mu\mu})^2]\frac{G_1^2}{8\pi}\frac{f_K^2 M_K^5}{(m_d + m_s)^2}. \tag{9}$$

Here extra factor of 2 occurs because the amplitudes $A(K^0 \to \mu^+\mu^-) = A(\bar{K^0} \to \mu^+\mu^-)$ and $K_L = \frac{K^0 + \bar{K^0}}{\sqrt{2}}$. We see that $G_1^2[(g_S^{\mu\mu})^2 + (g_P^{\mu\mu})^2]$ can be calculated from $\Gamma(K_L \to \mu^+\mu^-)$, once we know the value of $(g_P^{sd})^2$. In order to determine the value of $(g_P^{sd})^2$, we consider $K^0 - \bar{K^0}$ mixing. The effective scalar/pseudoscalar new physics lagrangian for this process can be obtained from that of $\bar{s} \to \bar{d}\mu^+\mu^-$ by replacing lepton current by corresponding quark current or equaivalently from effective lagrangian of eq. (5) where $b - s$ quark bilinear is replaced by $s - d$ quark bilinear,

$$L^{SP}_{K^0 - \bar{K^0}} = G_2\, \bar{s}(g_S^{sd} + g_P^{sd}\gamma_5)d\ \bar{s}(g_S^{sd} + g_P^{sd}\gamma_5)d. \tag{10}$$

From this lagrangian, we obtain the $K_L - K_S$ mass difference to be

$$\Delta m_K = \frac{1}{2M_K} G_2 \, (g_P^{ds})^2 \hat{B}_K \frac{f_K^2 M_K^4}{(m_s + m_d)^2}. \tag{11}$$

Thus the effective $s - d$ pseudoscalar coupling is

$$(g_P^{sd})^2 = \frac{2\Delta m_K (m_d + m_s)^2}{\hat{B_K} f_K^2 M_K^3 G_2}. \tag{12}$$

Substituting the above value of $(g_P^{sd})^2$ in eq. (9), we get

$$G_1^2[(g_S^{\mu\mu})^2 + (g_P^{\mu\mu})^2] = \frac{2\pi G_2 \hat{B_K}}{M_K^2 \Delta m_K} \Gamma_{NP}(K_L \to \mu^+\mu^-). \tag{13}$$

Substituting the value of $G_1^2[(g_S^{\mu\mu})^2 + (g_P^{\mu\mu})^2]$ from eq. (13) and $(g_P^{sb})^2$ from eq. (7) in eq. (4), we get

$$\Gamma_{NP}(B_s \to \mu^+\mu^-) = \frac{1}{2} \left(\frac{M_{B_s}}{M_K}\right)^2 \left(\frac{\Delta m_{B_s}}{\Delta m_K}\right) \left(\frac{\hat{B_K}}{\hat{B}_{B_s}}\right) \Gamma_{NP}(K_L \to \mu^+\mu^-). \tag{14}$$

The branching ratio is given by,

$$B_{NP}(B_s \to \mu^+\mu^-) = \frac{1}{2} \left(\frac{M_{B_s}}{M_K}\right)^2 \left(\frac{\Delta m_{B_s}}{\Delta m_K}\right) \left(\frac{\hat{B_K}}{\hat{B}_{B_s}}\right) \left[\frac{\tau(B_s)}{\tau(K_L)}\right] B_{NP}(K_L \to \mu^+\mu^-). \tag{15}$$

We wish to obtain the largest possible value for $B(B_s \to \mu^+\mu^-)$. To this end, we make the liberal assumption that the experimental values of Δm_{B_s}, Δm_K and $B_{NP}(K_L \to \mu^+\mu^-)$ are saturated by new physics couplings. The decay rate for $K_L \to \mu^+\mu^-$ consists of both long distance and short distance contributions. The new physics we consider here, contributes only to the short distance part of the decay rate. In ref [14], an upper limit on the short distance contribution to $B(K_L \to \mu^+\mu^-)$ is calculated to be 2.5×10^{-9}. The mass difference of the B_s mesons is recenly measured by the CDF collaboration to be $\Delta m_{B_s} = (1.17 \pm 0.01) \times 10^{-11}\, GeV$ [13]. The bag parameters for the K and the B_s mesons are $\hat{B_K} = (0.58 \pm 0.04)$ and $\hat{B}_{B_s} = (1.30 \pm 0.10)$ [15]. The values of the other parameters of eq. (15) are taken from Review of Particle Properties [16]: $\Delta m_K = (3.48 \pm 0.01) \times 10^{-15}\, GeV$ $\tau(B_s) = (1.47 \pm 0.06) \times 10^{-12}\, Sec$ and $\tau(K_L) = (5.11 \pm 0.02) \times 10^{-8}\, Sec.$ Substituting these values in eq. (15), we get

$$B_{NP}(B_s \to \mu^+\mu^-) = (6.30 \pm 0.75) \times 10^{-9}, \tag{16}$$

where all the errors are added in quadrature. At 3σ, $B_{SM}(B_s \to \mu^+\mu^-) < 7.7 \times 10^{-9}$ where as $B_{NP}(B_s \to \mu^+\mu^-) < 8.55 \times 10^{-9}$. Thus we see that this upper bound is almost the same as the SM prediction even if we maximize the new physics couplings by assuming that they

saturate the experimental values. Therefore the present data on $B_s - \bar{B}_s$ mixing together with data on $K^0 - \bar{K^0}$ mixing and $K_L \to \mu^+\mu^-$ decay puts a strong constraint on new physics scalar/pseudoscalar couplings and doesn't allow a large boost in the branching ratio of $B_s \to \mu^+\mu^-$.

We now assume that the new physics involving scalar/pseudoscalar couplings accounts for the difference between the experimental values and the SM predictions of Δm_K, Δm_{B_s} and the short distance contribution to $\Gamma(K_L \to \mu^+\mu^-)$. The SM value for $B_s - \bar{B}_s$ is given by [17, 18],

$$(\Delta m_{B_s})_{SM} = \frac{G_F^2}{6\pi^2}\eta_B M_{B_s}\left(\hat{B}_{B_s} f_{B_s}^2\right) M_W^2 S(x_t)\left|V_{ts}\right|^2 \;=\; (1.16 \pm 0.32) \times 10^{-11}\, GeV, \qquad (17)$$

with $f_{B_s}\sqrt{\hat{B}_{B_s}} = (262 \pm 35)\, MeV$ [15] , $\eta_B = 0.55 \pm 0.01$[18] and $|V_{ts}| = 0.0409 \pm 0.0009$ [16]. $S(x_t)$ with $x_t = m_t^2/m_W^2$ is one of the Inami-Lim functions [19]. The SM value for $K^0 - \bar{K^0}$ mixing is given by [20],

$$(\Delta m_K)_{SM} = \frac{G_F^2}{6\pi^2}\left(\hat{B}_K f_K^2\right) M_K M_W^2 \left[\lambda_c^{*2}\eta_1 S(x_c) + \lambda_t^{*2}\eta_2 S(x_t) + 2\lambda_c^*\lambda_t^*\eta_3 S(x_c, x_t)\right], \qquad (18)$$

where $\lambda_j = V_{js}^* V_{jd}$, $x_j = m_j^2/m_W^2$. The functions S are given by [21, 22],

$$S(x_t) = 2.46\left(\frac{m_t}{170\, GeV}\right)^2, \quad S(x_c) = x_c. \qquad (19)$$

$$S(x_c, x_t) = x_c\left[\ln\frac{x_t}{x_c} - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2 \ln x_t}{4(1-x_t)^2}\right]. \qquad (20)$$

Using $\eta_1 = (1.32 \pm 0.32)$ [23], $\eta_2 = (0.57 \pm 0.01)$ [18], $\eta_3 = (0.47 \pm 0.05)$ [24, 25], $\hat{B_K} = (0.58 \pm 0.04)$ [15] ; $f_K = (159.8 \pm 1.5)\, MeV$, $|V_{cs}| = 0.957 \pm 0.017 \pm 0.093$, $|V_{cd}| = 0.230 \pm 0.011$, $|V_{ts}| = 0.0409 \pm 0.0009$ and $|V_{td}| = 0.0074 \pm 0.0008$ [16], we get

$$(\Delta m_K)_{SM} = (1.87 \pm 0.49) \times 10^{-15}\, GeV. \qquad (21)$$

All the masses were taken from [16]. Considering only the short-distance effects, the SM branching ratio for $K_L \to \mu^+\mu^-$ in next-to-next-to-leading order of QCD is $(0.79 \pm 0.12) \times 10^{-9}$ [26]. Substracting out the SM contribution from the experimental values of Δm_{B_s}, Δm_K and $B_{NP}(K_L \to \mu^+\mu^-)$, we get

$$\begin{aligned} B_{NP}(B_s \to \mu^+\mu^-) \;=\; & \left[\frac{(\Delta m_{B_s})_{exp} - (\Delta m_{B_s})_{SM}}{(\Delta m_K)_{exp} - (\Delta m_K)_{SM}}\right]\frac{1}{2}\left(\frac{M_{B_s}}{M_K}\right)^2\left(\frac{\hat{B_K}}{\hat{B}_{B_s}}\right)\left[\frac{\tau(B_s)}{\tau(K_L)}\right] \\ & \left(B_{exp}(K_L \to \mu^+\mu^-)_{short} - B_{SM}(K_L \to \mu^+\mu^-)\right). \qquad (22) \end{aligned}$$

Substituting the experimental values and the SM predictions in the above equation, and adding all the errors in quadrature, we get

$$B_{NP}(B_s \to \mu^+\mu^-) = (0.08 \pm 2.54) \times 10^{-9}. \tag{23}$$

which is consistent with zero. At 3σ, the upper limit on the new physics contribution is close to SM prediction. Thus the present data on Δm_{B_s} along with Δm_K and $B_{NP}(K_L \to \mu^+\mu^-)$ puts strong constraints on new physics scalar/pseudoscalar couplings and doesn't allow a large enhancement in the branching ratio of $B_{NP}(B_s \to \mu^+\mu^-)$ much beyond the SM predictions. ***New physics at most can cause a factor of two enhancement but not an order of magnitude***. Hence the total branching ratio which is the sum of SM contribution and new physics contribution will be of the order of 10^{-8} and hence reachable at LHC.

Conclusions:

In this letter, we considered the constraints on the New Physics couplings of scalar/pseudoscalar type in the $b \to s$ transition. It was shown previously that only such New Physics can give rise to an order of magnitude enhancement of the decay rate for $B_s \to \mu^+\mu^-$. Using the recent data on $B_s - \bar{B}_s$ mixing, together with the data on $K^0 - \bar{K}^0$ mixing and the short distance contribution to $K_L \to \mu^+\mu^-$), we obtained very strong bounds on $B(B_s \to \mu^+\mu^-)$. New Physics in the form of scalar/pseudoscalar couplings can at most increase the $B(B_s \to \mu^+\mu^-)$ by a factor of 2 compared to its Standard Model prediction. An order magnitude enhancement, previously allowed, is ruled out.

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